Demana HO wk13 Tues

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Chapter 6 Review Exercises

In Exercises 1–8, evaluate the integral analytically. Then use NINT to support your result.

1. $\int_0^{\pi/3} \sec^2 \theta \, d\theta$

2. $\int_1^2 \left(x + \frac{1}{x^2}\right) dx$

3. $\int_0^1 \frac{36\, dx}{(2x+1)^3}$

4. $\int_{-1}^1 2x \sin(1 - x^2)\, dx$

5. $\int_0^{\pi/2} 5 \sin^{3/2} x \cos x \, dx$

6. $\int_{1/2}^4 \frac{x^2 + 3x}{x}\, dx$

7. $\int_0^{\pi/4} e^{\tan x} \sec^2 x \, dx$

8. $\int_1^e \frac{\sqrt{\ln r}}{r}\, dr$

In Exercises 9–20, evaluate the integral.

9. $\int \frac{\cos x}{2 - \sin x}\, dx$

10. $\int \frac{dx}{\sqrt[3]{3x+4}}$

11. $\int \frac{t\, dt}{t^2 + 5}$

12. $\int \frac{1}{\theta^2} \sec \frac{1}{\theta} \tan \frac{1}{\theta}\, d\theta$

13. $\int \frac{\tan(\ln y)}{y}\, dy$

14. $\int e^x \sec(e^x)\, dx$

15. $\int \frac{dx}{x \ln x}$

16. $\int \frac{dt}{t\sqrt{t}}$

17. $\int x^3 \cos x \, dx$

18. $\int x^4 \ln x \, dx$

19. $\int e^{3x} \sin x \, dx$

20. $\int x^2 e^{-3x}\, dx$

In Exercises 21–28, solve the initial value problem analytically. Support your solution by overlaying its graph on a slope field of the differential equation.

21. $\frac{dy}{dx} = 1 + x + \frac{x^2}{2}, \quad y(0) = 1$

22. $\frac{dy}{dx} = \left(x + \frac{1}{x}\right)^2, \quad y(1) = 1$

23. $\frac{dy}{dt} = \frac{1}{t+4}, \quad y(-3) = 2$

24. $\frac{dy}{d\theta} = \csc 2\theta \cot 2\theta, \quad y(\pi/4) = 1$

25. $\frac{d^2y}{dx^2} = 2x - \frac{1}{x^2}, \quad x > 0, \quad y'(1) = 1, \quad y(1) = 0$

26. $\frac{d^3r}{dt^3} = -\cos t, \quad r''(0) = r'(0) = r(0) = -1$

27. $\frac{dy}{dx} = y + 2, \quad y(0) = 2$

28. $\frac{dy}{dx} = (2x+1)(y+1), \quad y(-1) = 1$

In Exercises 29–32, assume that $1 - \sqrt{x}$ is an antiderivative of f and $x + 2$ is an antiderivative of g. Find the indefinite integral of the function.

29. $-f(x)$

30. $x + f(x)$

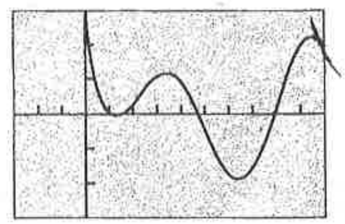
31. $2f(x) - g(x)$

32. $g(x) - 4$

In Exercises 33 and 34, match the indefinite integral with the graph of one of the antiderivatives of the integrand.

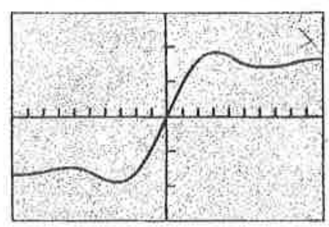
33. $\int \frac{\sin x}{x}\, dx$

34. $\int e^{-x^2}\, dx$



[–3, 10] by [–3, 3]

(a)



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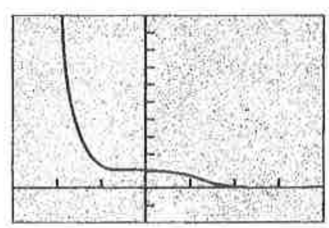
(b)



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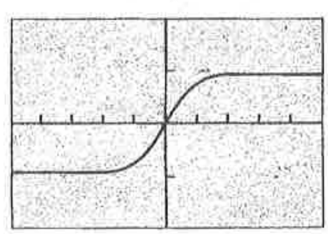
(c)



[–5, 5] by [–2, 2]

(d)

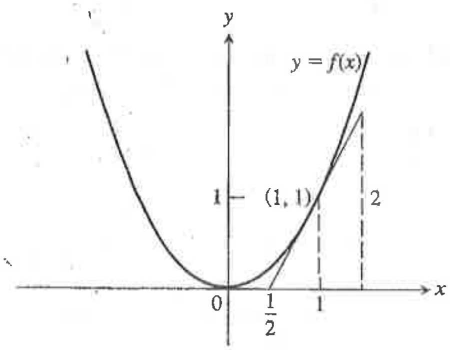
35. **Writing to Learn** The figure shows the graph of the function $y = f(x)$ that is the solution of one of the following initial value problems. Which one? How do you know?

i. $dy/dx = 2x,\quad y(1) = 0$

ii. $dy/dx = x^2,\quad y(1) = 1$

iii. $dy/dx = 2x + 2,\quad y(1) = 1$

iv. $dy/dx = 2x,\quad y(1) = 1$



36. **Writing to Learn** Does the following initial value problem have a solution? Explain.

$$\frac{d^2y}{dx^2} = 0,\quad y'(0) = 1,\quad y(0) = 0$$

37. *Moving Particle* The acceleration of a particle moving along a coordinate line is

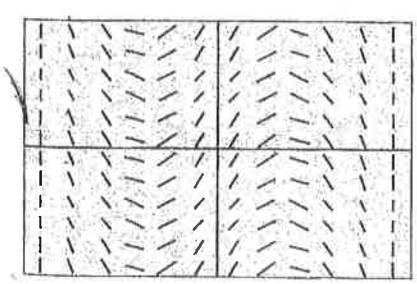
$$\frac{d^2s}{dt^2} = 2 + 6t \text{ m/sec}^2.$$

At $t = 0$ the velocity is 4 m/sec.

(a) Find the velocity as a function of time t.

(b) How far does the particle move during the first second of its trip, from $t = 0$ to $t = 1$?

38. *Sketching Solutions* Draw a possible graph for the function $y = f(x)$ with slope field given in the figure that satisfies the initial condition $y(0) = 0$.



[–10, 10] by [–10, 10]

In Exercises 39 and 40, use the stated method to solve the initial value problem on the given interval starting at x_0 with $dx = 0.1$.

39. Euler; $y' = y + \cos x,\quad y(0) = 0;\quad 0 \le x \le 2;\quad x_0 = 0$

40. improved Euler; $y' = (2 - y)(2x + 3),\quad y(-3) = 1;\quad -3 \le x \le -1;\quad x_0 = -3$

In Exercises 41 and 42, use the stated method with $dx = 0.05$ to estimate $y(c)$ where y is the solution to the given initial value problem.

41. improved Euler; $c = 3;\quad \dfrac{dy}{dx} = \dfrac{x - 2y}{x + 1},\quad y(0) = 1$

42. Euler; $c = 4;\quad \dfrac{dy}{dx} = \dfrac{x^2 - 2y + 1}{x},\quad y(1) = 1$

In Exercises 43 and 44, use the stated method to solve the initial value problem graphically, starting at $x_0 = 0$ with (a) $dx = 0.1$ and (b) $dx = -0.1$.

43. Euler; $\dfrac{dy}{dx} = \dfrac{1}{e^{x+y+2}},\quad y(0) = -2$

44. improved Euler; $\dfrac{dy}{dx} = -\dfrac{x^2 + y}{e^y + x},\quad y(0) = 0$

45. *Californium-252* What costs \$27 million per gram and can be used to treat brain cancer, analyze coal for its sulfur content, and detect explosives in luggage? The answer is californium-252, a radioactive isotope so rare that only about 8 g of it have been made in the western world since its discovery by Glenn Seaborg in 1950. The half-life of the isotope is 2.645 years—long enough for a useful service life and short enough to have a high radioactivity per unit mass. One microgram of the isotope releases 170 million neutrons per second.

(a) What is the value of k in the decay equation for this isotope?

(b) What is the isotope's mean life? (See Exercise 17, Section 6.4.)

46. *Cooling a Pie* A deep-dish apple pie, whose internal temperature was 220°F when removed from the oven, was set out on a 40°F breezy porch to cool. Fifteen minutes later, the pie's internal temperature was 180°F. How long did it take the pie to cool from there to 70°F?

47. *Finding Temperature* A pan of warm water (46°C) was put into a refrigerator. Ten minutes later, the water's temperature was 39°C; 10 minutes after that, it was 33°C. Use Newton's Law of Cooling to estimate how cold the refrigerator was.

48. *Art Forgery* A painting attributed to Vermeer (1632–1675), which should contain no more than 96.2% of its original carbon-14, contains 99.5% instead. About how old is the forgery?

49. *Carbon-14* What is the age of a sample of charcoal in which 90% of the carbon-14 that was originally present has decayed?

50. *Appreciation* A violin made in 1785 by John Betts, one of England's finest violin makers, cost \$250 in 1924 and sold for \$7500 in 1988. Assuming a constant relative rate of appreciation, what was that rate?